Period 3, Oct 16, 2024

$$\lim_{h \to 0} \frac{F(x+h) - F(x)}{h} = F'(x)$$

13. $\sin x = x(1 + \tan y)$

$$\cos x = 1(1+\tan y) + x(0 + \sec^2 y \frac{dy}{dx})$$

$$\cos x = 1 + \tan y + x \sec^2 y \frac{dy}{dx}$$

$-1 \quad -1 - \tan y$

$- \tan y$

$$\frac{\cos x - 1 - \tan y}{x \sec^2 y} = \frac{x \sec^2 y}{x \sec^2 y} \frac{dy}{dx}$$

$$\frac{\cos x - 1 - \tan y}{x \sec^2 y} = \frac{dy}{dx}$$

5. $x^3 - xy + y^2 = 7$

$$3x^2 - 1 \cdot y + (-x) \cdot \frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$3x^2 - y - x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$-3x^2 + y \qquad -3x^2 + y$

$$\frac{\frac{dy}{dx}(-x+2y)}{-x+2y} = \frac{y - 3x^2}{-x+2y}$$

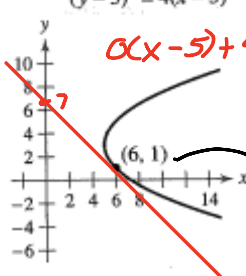
$$\frac{dy}{dx} = \frac{(y-3x^2) \cdot -1}{(-x+2y) \cdot -1} = \frac{3x^2 - y}{x - 2y}$$

Famous Curves **In Exercises 33–40, find an equation of the tangent line to the graph at the given point. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.**

33. Parabola

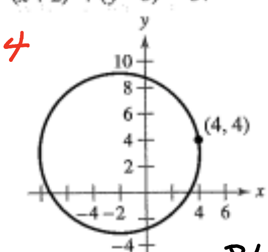
$(y-3)^2 = 4(x-5)$

$0(x-5) + 4(1) = 4$



34. Circle

$(x+2)^2 + (y-3)^2 = 37$



Plug

$$(y-3)^2 = 4x - 20 \Rightarrow 2(y-3)^1 \frac{dy}{dx} = 4$$

Plug in (6,1)

$$2(1-3)\frac{dy}{dx} = 4$$

$$2 \cdot -2 \frac{dy}{dx} = 4$$

$$-4\frac{dy}{dx} = 4$$

$$\frac{dy}{dx} = -1 = m = \text{Slope at } (6,1)$$

Line

$y - y_1 = m(x - x_1)$

$y - 1 = -1(x - 6)$

$y - 1 = -x + 6$

$+1 \qquad +1$

$y = -x + 7$

$L = (y-3)^2 \quad$ Find $\frac{dL}{dx}$

$u = y - 3$

$\frac{du}{dx} = \frac{dy}{dx} - 0$

$L = u^2$

$\frac{dL}{du} = 2u$

$\frac{du}{dx} \cdot \frac{dL}{du}$

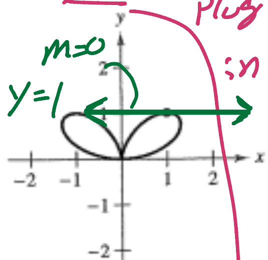
$\frac{dy}{dx} \cdot 2u = 2(y-3)\frac{dy}{dx}$

31. Bifolium:

$(x^2 + y^2)^2 = 4x^2y$

Point: (1, 1)

Plug in



32.

$L = (x^2+y^2)^2 \Rightarrow L = u^2$

Find $\frac{dL}{dx}$ $\quad \frac{dL}{du} = 2u$

$u = x^2+y^2$

$\frac{du}{dx} = 2x + 2y\frac{dy}{dx}$

$\frac{du}{dx} \cdot \frac{dL}{du} = \frac{dL}{dx}$

$\left(2x+2y\frac{dy}{dx}\right) \cdot 2(x^2+y^2) = \frac{dL}{dx}$

$(x^2+y^2)^2 = 4x^2y$

$\left(2x+2y\frac{dy}{dx}\right)\cdot 2(x^2+y^2) = 8x\cdot y + 4x^2 \cdot \frac{dy}{dx}$

$\left(2\cdot 1+2\cdot 1\frac{dy}{dx}\right)\cdot 2(1^2+1^2) = 8\cdot 1\cdot 1 + 4(1)^2\frac{dy}{dx}$

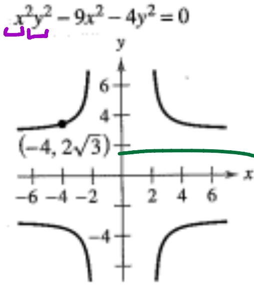
$4\left(2+2\frac{dy}{dx}\right) = 8 + 4\frac{dy}{dx} \Rightarrow 8 + 8\frac{dy}{dx} = 8 + 4\frac{dy}{dx}$

$-8 \qquad -8$

$8\frac{dy}{dx} = 4\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = 0 = m = 0$

37. Cruciform

$x^2y^2 - 9x^2 - 4y^2 = 0$



38.

Plug in

$y - 2\sqrt{3} = \frac{\sqrt{3}}{6}(x - (-4))$

Keep going

$\frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6} = \frac{dy}{dx}$

Point $(-4, 2\sqrt{3})$

$~~2xy^2~~ + x^2 \cdot 2y\frac{dy}{dx} - ~~18x~~ - 8y\frac{dy}{dx} = 0$

$-2xy^2 \qquad +18x \qquad -2xy^2 + 18x$

$\frac{dy}{dx}(x^2 2y - 8y) = \frac{-2xy^2 + 18x}{2x^2y - 8y}$

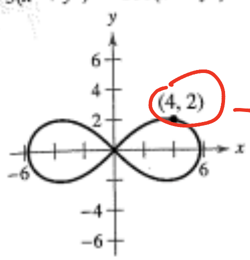
$2x^2y - 8y$

$\frac{dy}{dx} = \frac{-2(-4)(2\sqrt{3})^2 + 18(-4)}{2(-4)^2(2\sqrt{3}) - 8(2\sqrt{3})} = \frac{96 - 72}{64\sqrt{3} - 16\sqrt{3}} = \frac{24}{48\sqrt{3}} = \frac{1}{2\sqrt{3}}$

12

39. Lemniscate

$3(x^2+y^2)^2 = 100(x^2-y^2)$



$L = (x^2+y^2)^2 \Rightarrow L = u^2$

Find $\frac{dL}{dx}$ $\qquad \frac{dL}{du} = 2u$

$u = x^2+y^2$

$\frac{du}{dx} = 2x + 2y\frac{dy}{dx}$

$\frac{du}{dx} \cdot \frac{dL}{du} = \frac{dL}{dx} = \left(2x+2y\frac{dy}{dx}\right)\left(2(x^2+y^2)\right)$

$3(x^2+y^2)^2 = 100x^2 - 100y^2$

$3 \cdot 2\left(2x+2y\frac{dy}{dx}\right)(x^2+y^2) = 200x - 200y\frac{dy}{dx}$

$6\left(2x+2y\frac{dy}{dx}\right)(x^2+y^2) = 200x - 200y\frac{dy}{dx}$

Plug in $(4,2)$

$6\left(2\cdot 4 + 2\cdot 2\frac{dy}{dx}\right)(4^2+2^2) = 200(4) - 200\cdot 2\frac{dy}{dx}$

$6\left(8+4\frac{dy}{dx}\right)(20) = 800 - 400\frac{dy}{dx}$

$120\left(8+4\frac{dy}{dx}\right) = 800 - 400\frac{dy}{dx}$

$960 + 480\frac{dy}{dx} = 800 - 400\frac{dy}{dx}$

$-800 \quad -480\frac{dy}{dx} \quad -800 \quad -480\frac{dy}{dx}$

$\frac{160}{-880} = \frac{-880\frac{dy}{dx}}{-880} = \frac{80\cdot 2}{-80\cdot 11} = \frac{dy}{dx}$

$m = -\frac{2}{11}$

Slope $= -\frac{2}{11}$

Point $= (4,2)$

$y - 2 = -\frac{2}{11}(x-4)$

Keep going

$y = -\frac{2}{11}x + \frac{30}{11}$

Point $(-4, 2\sqrt{3})$

$m = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$

$y - 2\sqrt{3} = \frac{\sqrt{3}}{6}(x-(-4))$

$y - 2\sqrt{3} = \frac{\sqrt{3}}{6}(x+4)$

$y - 2\sqrt{3} = \frac{\sqrt{3}}{6}x + \frac{4\sqrt{3}}{6}$

$+2\sqrt{3} \qquad +2\sqrt{3}$

$\frac{4\sqrt{3}}{6} + \frac{2\sqrt{3}\cdot 6}{1\cdot 6}$ $\quad 12\sqrt{3}$

$y = \frac{\sqrt{3}}{6}x + \frac{16\sqrt{3}}{6} = \frac{\sqrt{3}}{6}x + \frac{8\sqrt{3}}{3}$

15. $y = \sin xy$

$L = \sin xy$ Find $\frac{dL}{dx}$

$\Rightarrow L = \sin u$

$\frac{dL}{du} = \cos u$

$u = x\,y$

$\frac{du}{dx} = 1\cdot y + x\cdot\frac{dy}{dx}$

$\frac{du}{dx}\cdot\frac{dL}{du} = \frac{dL}{dx}$

$(y + x\frac{dy}{dx})\cos u$

$(y + x\frac{dy}{dx})\cos xy$

$\frac{dy}{dx} = (y + x\frac{dy}{dx})\cos xy$

$\frac{dy}{dx} = y\cos xy + x\frac{dy}{dx}\cos xy$

$-x\frac{dy}{dx}\cos xy \qquad -x\frac{dy}{dx}\cos xy$

$\frac{dy}{dx} - x\frac{dy}{dx}\cos xy = y\cos xy$

$\frac{dy}{dx}(1 - x\cos xy) = y\cos xy$

$\overline{1 - x\cos xy} \qquad \overline{1 - x\cos xy}$

$\frac{dy}{dx} = \frac{y\cos xy}{1 - x\cos xy}$

---

25. $x^{2/3} + y^{2/3} = 5, \quad (8, 1)$

$\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}}\frac{dy}{dx} = 0$

Plug in Point

$\frac{2}{3\sqrt[3]{x}} + \frac{2}{3\sqrt[3]{y}}\frac{dy}{dx} = 0$

$\frac{2}{3\sqrt[3]{8}} + \frac{2}{3\sqrt[3]{1}}\frac{dy}{dx}$

$\frac{2}{3\cdot 2} + \frac{2}{3\cdot 1}\frac{dy}{dx} = 0$

$\frac{1}{3} + \frac{2}{3}\frac{dy}{dx} = 0 \Rightarrow \frac{2}{3}\frac{dy}{dx} = -\frac{1}{3}\cdot\frac{3}{2}$

$-\frac{1}{3} \qquad -\frac{1}{3}$

$\frac{dy}{dx} = -\frac{1}{2}$

23. $y^2 = \frac{x^2 - 49}{x^2 + 49}, \quad (7, 0)$

$2y\frac{dy}{dx} = \frac{2x(x^2+49) - (x^2-49)(2x)}{(x^2+49)^2}$

$2y\frac{dy}{dx} = \frac{2x^3 + 98x - 2x^3 + 98x}{(x^2+49)^2}$

$2y\frac{dy}{dx} = \frac{196x}{(x^2+49)^2} = 0$ Plug in (7,0)

$\frac{dy}{dx} = \frac{196(7)}{2(0)(7^2+49)^2} = \varnothing$

In Exercises 45–50, find d^2y/dx^2 in terms of x and y.

45. $x^2 + y^2 = 4$

46. $x^2y^2 - 2x = 3$

47. $x^2 - y^2 = 36$

48. $1 - xy = x - y$

49. $y^2 = x^3$

50. $y^2 = 10x$

$$y^2 = x^3$$

$$2y\frac{dy}{dx} = 3x^2$$

$$\frac{dy}{dx} = \frac{3x^2}{2y}$$

plug in

$$\frac{d^2y}{dx^2} = \frac{6x(2y) - 3x^2 \cdot 2\frac{dy}{dx}}{(2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{12xy - 6x^2\left(\frac{3x^2}{2y}\right)}{4y^2}$$

$$\frac{d^2y}{dx^2} = \frac{12xy - \frac{18x^4}{2y}}{4y^2} = \frac{\frac{12xy \cdot y}{1 \cdot y} - \frac{9x^4}{y}}{4y^2}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{12xy^2}{y} - \frac{9x^4}{y}}{4y^2} = \frac{\frac{12xy^2 - 9x^4}{y}}{\frac{4y^2}{1}}$$

$$\frac{d^2y}{dx^2} = \frac{12xy^2 - 9x^4}{y} \cdot \frac{1}{4y^2} = \frac{12xy^2 - 9x^4}{4y^3}$$

$$\frac{d^2y}{dx^2} = \frac{12xy^2 - 9x \cdot x^3}{4y^3} = \frac{12xy^2 - 9xy^2}{4y^3} = \frac{3xy^2}{4y^3}$$

---

45. $x^2 + y^2 = 4$

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$$

$$\frac{d^2y}{dx^2} = \frac{-1 \cdot y - (-x) \cdot \frac{dy}{dx}}{y^2} =$$

$$\frac{d^2y}{dx^2} = \frac{-y + x \cdot \frac{-x}{y}}{y^2} = \frac{\frac{-y \cdot y}{1 \cdot y} - \frac{x^2}{y}}{y^2} = \frac{\frac{-y^2 - x^2}{y}}{\frac{y^2}{1}}$$

$$\frac{d^2y}{dx^2} = \frac{-y^2 - x^2}{y} \cdot \frac{1}{y^2} = \frac{-1(x^2 + y^2)}{y^3} = \frac{-1 \cdot 4}{y^3}$$

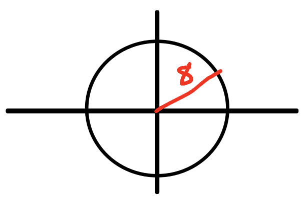
In Exercises 17–20, (a) find two explicit functions by solving the equation for y in terms of x, (b) sketch the graph of the equation and label the parts given by the corresponding explicit functions, (c) differentiate the explicit functions, and (d) find dy/dx implicitly and show that the result is equivalent to that of part (c).

17. $x^2 + y^2 = 64$ **18.** $x^2 + y^2 - 4x + 6y + 9 = 0$

$2x + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$

$\frac{-x}{\pm\sqrt{64-x^2}}$



$x^2 + y^2 = 64$

$\sqrt{y^2} = \sqrt{64 - x^2}$

$y = \pm\sqrt{64 - x^2}$

$y = \pm(64 - x^2)^{\frac{1}{2}}$ $\Rightarrow y = \pm u^{\frac{1}{2}}$

$u = 64 - x^2$

$\frac{du}{dx} = -2x$ $\frac{dy}{du} = \frac{\pm 1}{2\sqrt{u}}$

$\frac{du}{dx} \cdot \frac{dy}{du} = -2x \cdot \frac{\pm 1}{2\sqrt{64-x^2}}$

$\frac{dy}{dx} = \frac{\pm 2x}{2\sqrt{64-x^2}} = \frac{\pm 2x}{2y}$